Energy Quantization from Schwarzschild Anti-de Sitter Black Hole

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Abstract

In this present work we address the study of Bekenstein-Hawking entropy of Schwarzschild Anti-de Sitter (SAdS) black hole by using energy quantization method like Bohr's atomic model. We have also shown that the change of entropy as well as purely thermal emission rate is dependent of quantum number and come close to zero for large quantum number.

Keywords: Energy Quantization, SAdS black hole, Circular orbit, quantum number, gravitoelectromagnetism.

1. Introduction

In recent times, measuring black holes is a crucial topic of research in quantum gravity theory of physics[1]. But no agreeable solution has yet been found for quantization of black holes since the radius is theonly parameter of the black holes with no charge and angular momentum and as such it is difficult fora researcher to observe what occurs inside from outside. A researcher should consider all the conservedinformation on the surface, which is termed as event horizon of the black hole. First in 1973, Bekenstein[2] considered the black hole horizon area as an adiabatic invariant quantity which is proportional to thearea spectrum and documented as black hole entropy [2, 3, 4, 5, 6]. The most raveling truth of black holeradiation [7, 8] was discovered by Stephen Hawking in 1975 and since then quite a few research studies havebeen made to determine this quantum effect [9]. Currently, the radiation of black holes is called 'Hawkingradiation'. The entropic structure given in Ref. [10, 11] undeniably supports new thought on quantumproperties of gravity beyond conventional physics. However, the quantization of gravity revealed in thispaper should not be interpreted only a support of quantization of black hole as an entropic forces [11, 12, 13],it also recommends a new approach of unifying gravity with quantum theory. Thus the present study willlead to a new understanding and outlook regarding gravity of a black hole. In 1931, Dirac proved thatthe existence of magnetic monopoles lead to quantization of electric charge [14]. Alike to the Dirac theory,Zee [15] presented a new gravitational analog of Dirac quantization condition in 1985, which is popularlyknown as the theory of gravitoelectromagnetism(GEM) [16]. In this GEM analogy, the electric charge andthe electric field of Maxwell electromagnetic theory play the role of the mass of the test particle and thegravitational acceleration respectively. In GEM theory, the source of magnetic field is considered as thesubstance of current density in agreement with the Biot-Savart law and is called as GSM magnetic fieldwhich is a divergenceless quantity all over the world. In this GEM analogy, Zee [15] has considered theexistence of a gravitipole following Dirac. Due to the hypothetical nature of the gravitipole one can splitthe upper bound of energy without the quantization effects on energy level splitting in atoms and molecules[15]. One can also quantize the mass of the text particle by intensifying the action of the test particle.
In this study, we have used the method of Simanek [17] to find out the Hawking purely thermal emissionrate with the Bekenstein-Hawking Entropy by quantizing the energy having the gravitational field of SdSblack hole, which is a non-asymptotically flat solutions to the Einstein-Maxwell-Dilaton- Axion (EMDA)theory in 4-dimensions. We have quantized the energy of a test

ISSN 2229-5518

particle orbiting SdS black hole from thequantization of angular momentum. It is well-known for the spherically symmetric black hole that thecanonical formulation can be developed to study quantization by offering a foliation in the radial parametersince it is only a function of the Lagrangian coordinates. Although many works on Hawking radiation,entropic force, and higher dimensions have been done [18, 19, 20, 21, 22, 23, 24, 25, 26, 27], the quantizationof a black hole or it gravity is very limited and as such, there is ample scope for further study.Conversely, Zhang and Zhao was first proposed Hawking radiation of black hole from massive unchargedparticle tunneling [28] and charged particle tunneling [29,30]. Developing this work, a few researches havebeen carried out as charged particle tunneling [31, 32, 33]. Recently, Kerner and Mann expanded quantumtunneling methods for analyzing the temperature of Taub-NUT black holes [34]. Considering Kerner andMann's process Chen, Zu and Yang reformed Hamilton-Jacobi method for massive particle tunneling andinvestigate the Hawking radiation of the Taub-NUT black hole [35]. Using this method Hawking radiationof Kerr-NUT black hole [36], the charged black hole with a global monopole [37] and Schwarzschild-de Sitterblack hole [38] have been reviewed. Apply our previous study SdS black hole [39], we investigate the Hawkingradiation and energy quantization for SAdS black hole.
The plan of this paper is as follows: In Section 2 we investigate the Lagrangian and canonical momentafor SAdS black hole. We calculate the effective potential for radial motion of a Schwarzschild-anti-de Sittersolution in Section 3. We use these results in Section 4 to study the quantization of energy and Hawkingtemperature. In Section 5, we investigate the microcanonical ensemble.

2. Investigate the Lagrangian and canonical momenta of SAdSblack hole

The Schwarzschild Anti-de Sitter black hole with mass M and a negative cosmological constant constant ($\Lambda = 3/\ell^2$) can be configured as [38]

$$ds^2 = -\left(1-\frac{2M}{r}+\frac{r^2}{\ell^2}\right)c^2dt^2+\left(1-\frac{2M}{r}+\frac{r^2}{\ell^2}\right)^{-1}dr^2 + r^2\left(d\theta^2+\sin^2\theta\ d\varphi^2\right). \quad (1)$$

The coordinates are defined such that $-\infty \le t \le \infty, r \ge 0, 0 \le \theta \le \pi,$ and $0 \le \theta \le 2\pi$. The lapse function vanished at the zeros of the cubic equation $r^3+\ell^2 r - 2M\ell^2 = 0$. Solving this we get the real root of the form as [38]

$$r_{SAdS} = 2M\left(1-\frac{4M^2}{\ell^2}+\cdots\right). \quad (2)$$

When $\left(1-\frac{4M^2}{\ell^2}+\cdots\right) > 1$which indicate that the SAdS black hole radius is larger than SchwarzschildBlack hole ($r_s = 2M$). For the simplicity we can modify the Eq. (1) of the following form

$$ds^2 = -\left(1-\frac{2M}{r}\left(1-\frac{r^3}{2M\ell^2}\right)\right)c^2dt^2+\left(1-\frac{2M}{r}\left(1-\frac{r^3}{2M\ell^2}\right)\right)^{-1}dr^2 + r^2\left(d\theta^2+\sin^2\theta\ d\varphi^2\right). \quad (3)$$

For the first approximation, using $r_0 = 2M$ into the above matric (3) we have

$$ds^2 = -\left(1-\frac{2M}{r}\left(1-\frac{4M^2}{\ell^2}\right)\right)c^2dt^2+\left(1-\frac{2M}{r}\left(1-\frac{4M^2}{\ell^2}\right)\right)^{-1}dr^2 + r^2\left(d\theta^2+\sin^2\theta\ d\varphi^2\right). \quad (4)$$

Let us consider a test particle of mass m orbiting along the circular geodesics in the equatorial plane aroundSAdS black hole. Then according to the Ref. [40, 41] the above metric can be written of the form as

$$\mathrm{d}s^2 = -\left(1-\frac{2M}{r}\left(1-\frac{4M^2}{\ell^2}\right)\right)c^2dt^2 + \left(1-\frac{2M}{r}\left(1-\frac{4M^2}{\ell^2}\right)\right)^{-1}dr^2 + r^2d\varphi^2. \qquad (5)$$

If we take the black hole mass is larger than the Planck mass so that the Compton radius, $r_c = \hbar/Mc$ is verysmallest than the radius of SAdS black hole r_{SAdS}. For this reason the quantum fluctuations of the blackhole disregards [42]. The Lagrangian of the test particle in terms of the metric components g_{ij} is defined as

$$\mathcal{L} = g_{ij}\dot{x}^i\dot{x}^j$$
$$= -\frac{m}{2}\left[\left(1-\frac{2M}{r}\left(1-\frac{4M^2}{\ell^2}\right)\right)c^2\dot{t}^2 + \left(1-\frac{2M}{r}\left(1-\frac{4M^2}{\ell^2}\right)\right)^{-1}\dot{r}^2 + r^2\sin^2\theta\,\dot{\varphi}^2 + r^2\dot{\theta}^2\right]. \qquad (6)$$

While the SAdS spacetime is static and spherically symmetric, there exist two constants of motion for thetest particles, associated with two Killing vectors in terms of energy E and angular momentum L and the other two components can be written with the help of the canonical momenta $p_\alpha = \frac{\partial\mathcal{L}}{\partial\dot{x}^\alpha}$, defined as

$$E = \frac{1}{2}\frac{\partial\mathcal{L}}{\partial\dot{t}} = \square\square^2\left(1-\frac{2\square}{\square}\left(1-\frac{4M^2}{\ell^2}\right)\right)\dot{t};\; L = \frac{1}{2}\frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = mr^2\sin^2\theta\,\dot{\varphi}. \qquad (7)$$

$$p_r = \frac{\partial\mathcal{L}}{\partial\dot{r}} = mc^2\left(1-\frac{2M}{r}\left(1-\frac{4M^2}{\ell^2}\right)\right)^{-1}\dot{r};\; p_\theta = \frac{\partial\mathcal{L}}{\partial\dot{\theta}} = mr^2\dot{\theta}. \qquad (8)$$

3. Quantization of Circular Orbit due to Radial motion and Effectivepotential

The radial motion of a geodesic can be written as

$$g^{\alpha\alpha}p_\alpha^2 + m^2c^2 = 0, \qquad (9)$$

where $\alpha = 0, r, \theta, \phi$, and $(p_0 = E/c, \boldsymbol{p})$ expresses the magnitude of the energy-momentum. Inserting Eqs.(7-8) and using $\dot{\theta}^2 = 0$ and $\sin^2\theta = 1$the above Eq. (9), we have

$$\frac{E^2}{c^2}\left(1-\frac{2M}{r}\left(1-\frac{4M^2}{\ell^2}\right)\right)^{-1} + m^2\left(1-\frac{2M}{r}\left(1-\frac{4M^2}{\ell^2}\right)\right)^{-1}\dot{r}^2 + \frac{L^2}{r^2} + m^2c^2 = 0. \qquad (10)$$

In Ref. [17], we get the energy and angular momentum of the test particle of the form $\tilde{E} = \frac{E}{m}$, $\tilde{L} = \frac{L}{m}$. (11)

Using the above relation into Eq. (10) we get

$$m^2c^2\left[-\frac{\tilde{E}^2}{c^2}\left(1-\frac{2M}{r}\left(1-\frac{4M^2}{\ell^2}\right)\right)^{-1} + \frac{1}{c^2}\left(1-\frac{2M}{r}\left(1-\frac{4M^2}{\ell^2}\right)\right)^{-1}\dot{r}^2 + \frac{\tilde{L}^2}{c^2r^2} + 1\right] = 0. \qquad (12)$$

For the purpose of radial motion of the test particle the above Eq. (12) can be written of the form as

$$\frac{\dot{r}^2}{2} = \frac{\tilde{E}^2}{2c^2} - \frac{1}{2}\left(\frac{\tilde{L}^2}{r^2} + c^2\right)\left(1 - \frac{2M}{r}\left(1-\frac{4M^2}{\ell^2}\right)\right). \qquad (13)$$

We can describe the velocity for the time like particle by the parameters energy $E$and angular momentumℓ. Thus we have the effective potential V_{eff} for the radial motion can be written of the form

$$V_{eff} = \frac{1}{2}\left(\frac{\tilde{L}^2}{r^2} + c^2\right)\left(1 - \frac{2M}{r}\left(1 - \frac{4M^2}{\ell^2}\right)\right). \tag{14}$$

For the purpose of maximum potential, taking the first derivative of Eq. (14) with respect to the propertime and then equate to zero, we obtain as

$$\frac{c^2}{r^4}M\left(1 - \frac{2M}{r}\left(1 - \frac{4M^2}{\ell^2}\right)\right) \times \left(r^2 - \frac{\tilde{L}^2}{c^2 M\left(1 - \frac{4M^2}{\ell^2}\right)}r + \frac{3\tilde{L}^2}{c^2}\right) = 0. \tag{15}$$

Solving equation (15) we get the two roots of the form

$$R_{\pm} = \frac{\tilde{L}^2}{2c^2 M\left(1 - \frac{4M^2}{\ell^2}\right)} \pm \left[\left(\frac{\tilde{L}^2}{2c^2 M\left(1 - \frac{4M^2}{\ell^2}\right)}\right)^2 - \frac{3\tilde{L}^2}{c^2}\right]^{\frac{1}{2}}, \tag{16}$$

where R is the radius of the circular orbit. Simplifying Eq. (16) can be written as

$$R_{\pm} = \frac{\tilde{L}^2}{2c^2 M\left(1 - \frac{4M^2}{\ell^2}\right)} \times \left(1 \pm \left(1 - \frac{12c^2M^2\left(1 - \frac{4M^2}{\ell^2}\right)^2}{\tilde{L}^2}\right)^{\frac{1}{2}}\right). \tag{17}$$

From the Eq. (17) we observe that $R_{\pm}$ is real only when $\tilde{L}^2 \geq 12c^2M^2\left(1 - \frac{4M^2}{\ell^2}\right)^2$ and for the smallest stable orbit, the square root on the right hand side of Eq. (17) vanishes. Therefore, we must have

$$\tilde{L}^2 = 12c^2M^2\left(1 - \frac{4M^2}{\ell^2}\right)^2. \tag{18}$$

According to the conditions $\tilde{L}^2 \geq 12c^2M\left(1 - \frac{4M^2}{\ell^2}\right)^2$ and $\tilde{L}^2 >> 12c^2M\left(1 - \frac{4M^2}{\ell^2}\right)^2$ holds for large and largest stable circular orbits, respectively. Angular momentum can be quantized as a periodic function of time and help to quantize energy which idea develop by Wilson and Sommerfeld [43, 44] and those are closely related to the quantize angular momentum of the orbiting test particle. In order to quantize angular momentum J_ϕ with the help of canonical momentum L conjugate to the angular variable of the form

$$J_\phi = \int_0^{2\pi} L d\emptyset = n\,h. \tag{19}$$

Thus L is a constant of motion, Eq. (11) gives the quantization condition for the angular momentum of the form

$$L = m\tilde{L} = n\hbar, \quad \text{so that} \quad \tilde{L}_0 = n_0\hbar/m\,. \tag{20}$$

With the help of Compton radius and using Eq. (20) into the Eq. (18) becomes as

$$n_0^2 r_c^2 = 12M^2\left(1 - \frac{4M^2}{\ell^2}\right)^2. \tag{21}$$

The radius of the different stable circular orbit of the particle corresponds to n0 can be obtained by using Eq. (20) into Eq. (17) of the form

$$R_{+} = \left(\frac{n_0^2 r_c^2}{2M\left(1 - \frac{4M^2}{\ell^2}\right)}\right)\left(1 + \left(1 - \frac{12M^2\left(1 - \frac{4M^2}{\ell^2}\right)^2}{n_0^2 r_c^2}\right)^{\frac{1}{2}}\right). \tag{22}$$

Inserting Eq. (21) into Eq. (22), we get the approximate radius of the initial circular orbit R_0 as

$$R_0 \approx \left(\frac{n_0^2 r_c^2}{2M\left(1-\frac{4M^2}{\ell^2}\right)}\right). \tag{23}$$

In the limiting case when $\ell \to \infty$ this becomes $R_0 = n_0^2 \frac{r_c^2}{r_s}$and which agree with the result given in Ref. [17], where $r_s = 2M$ is the Schwarzschild radius. By replacing $n_1 = n_0 + 1$ in Eq. (22), we get the next higher circular orbit R_1 of the form

$$R_1 = (n_0+1)^2 \left(\frac{r_c^2}{2M\left(1-\frac{4M^2}{\ell^2}\right)}\right)\left(1+\left(1-\frac{12M^2\left(1-\frac{4M^2}{\ell^2}\right)^2}{r_c^2(n_0+1)^2}\right)^{\frac{1}{2}}\right). \tag{24}$$

For simplicity, let us consider $2M\left(1-\frac{4M^2}{\ell^2}\right) \gg r_c$ so that $n_0 >> 1$. We therefore can be written $(n_0+1)^2 = n_0^2 + 2n_0 + 1 = n_0^2\left[1+\frac{2}{n_0}+\frac{1}{n_0^2}\right] \approx n_0^2\left[1+\frac{2}{n_0}\right]$ and using this into Eq. (24) we get

$$R_1 = n_0^2\left(1+\frac{2}{n_0}\right)\left(\frac{r_c^2}{2M\left(1-\frac{4M^2}{\ell^2}\right)}\right)\left(1+\left(1-\frac{12M^2\left(1-\frac{4M^2}{\ell^2}\right)^2}{r_c^2(n_0+1)^2}\right)^{\frac{1}{2}}\right). \tag{25}$$

The parenthesis in the right side of the above equation can be approximated with the help of Eq. (21) ofthe form

$$1+\left(1-\frac{12M^2\left(1-\frac{4M^2}{\ell^2}\right)^2}{r_c^2(n_0+1)^2}\right)^{\frac{1}{2}} \approx 1+\left(1-\frac{1}{\left(1+\frac{2}{n_0}\right)^2}\right)^{\frac{1}{2}} = 1+\sqrt{\frac{2}{n_0}}. \tag{26}$$

Therefore, Eq. (25) reduced to

$$R_1 = R_0\left(1+\frac{2}{n_0}\right)\left(1+\sqrt{\frac{2}{n_0}}\right). \tag{27}$$

Proceeding in this way, the radius of the next higher circular orbit R_2 of the test particle can be evaluated from Eq. (22) of the form as

$$R_2 = R_0\left(1+\frac{4}{n_0}\right)\left(1+\sqrt{\frac{4}{n_0}}\right) = R_1\frac{\left(1+\frac{4}{n_0}\right)\left(1+\sqrt{\frac{4}{n_0}}\right)}{\left(1+\frac{2}{n_0}\right)\left(1+\sqrt{\frac{2}{n_0}}\right)}. \tag{28}$$

Proceeding in the similar way, the radius of the stable circular orbits of the particle can be written as

$$R_{n+1} = R_n\frac{\left(1+\frac{2n+2}{n_0}\right)\left(1+\sqrt{\frac{2n+2}{n_0}}\right)}{\left(1+\frac{2n}{n_0}\right)\left(1+\sqrt{\frac{2n}{n_0}}\right)}. \tag{29}$$

When $n_0 \to \infty$, we observe that $R_{n+1} = R_n$. Therefore, we may consummate that for large quantum number two nearby states coincide.

4.Quantized Energy

Our intent is to quantize the energy of the orbiting test particle with the help of angular momentum. Thus the equation (13) gives the energy for zero velocity at $r = R$ of the form as

$$\tilde{E}^2 = c^2\left(\frac{\tilde{L}^2}{R^2}+c^2\right)\left(1-\frac{2M\left(1-\frac{4M^2}{\ell^2}\right)}{R}\right). \tag{30}$$

But Eq. (15) gives at r = R

$$\frac{\tilde{L}^2}{R^2} = \frac{c^2}{\left(\frac{2R}{2M\left(1-\frac{4M^2}{\ell^2}\right)} - 3\right)}. \quad (31)$$

Using Eq. (11) and Eq. (31) into Eq. (30), we have

$$E^2 = m^2c^4\left(1 - \frac{2M\left(1-\frac{4M^2}{\ell^2}\right)}{R}\right)^2\left(1 - \frac{3M\left(1-\frac{4M^2}{\ell^2}\right)}{R}\right)^{-1}. \quad (32)$$

Therefore, the above equation can be written as

$$E = mc^2\left(1 - \frac{2M\left(1-\frac{4M^2}{\ell^2}\right)}{R}\right)\left(1 - \frac{3M\left(1-\frac{4M^2}{\ell^2}\right)}{R}\right)^{-1/2}. \quad (33)$$

Using the mechanical stability condition $\tilde{L}^2 = MGR$ in Eq. (17), the second term in the parenthesis of the Eq. (17) canbe written as

$$\frac{12c^2M^2\left(1-\frac{4M^2}{\ell^2}\right)^2}{\tilde{L}^2} = 3c^2\frac{2GM}{c^2}\frac{2M\left(1-\frac{4M^2}{\ell^2}\right)}{MGR}$$
$$= \frac{12M\left(1-\frac{4M^2}{\ell^2}\right)}{R}, \quad (34)$$

which gives with the help of Eq. (18) for the circular orbits corresponding to $n_0 >> 1$

$$\frac{12M\left(1-\frac{4M^2}{\ell^2}\right)}{R} << 1. \quad (35)$$

Therefore, Eq. (33) can be approximated to

$$E \approx mc^2\left(1 - \frac{M\left(1-\frac{4M^2}{\ell^2}\right)}{R}\right). \quad (36)$$

Applying Eq.(18) in Eq.(23) in the following form

$$2R \approx \left(\frac{n^2r_c^2}{M\left(1-\frac{4M^2}{\ell^2}\right)}\right) = \left(\frac{n^2\hbar^2}{m^2c^2M\left(1-\frac{4M^2}{\ell^2}\right)}\right) = \left(\frac{\tilde{L}^2}{c^2M\left(1-\frac{4M^2}{\ell^2}\right)}\right). \quad (37)$$

Substituting Eq. (37) into Eq. (36) we have

$$E \approx mc^2\left(1 - \frac{c^2M^2\left(1-\frac{4M^2}{\ell^2}\right)^2}{\tilde{L}^2}\right). \quad (38)$$

The nth quantized energy label E_n can be obtained of the form

$$E_n \approx mc^2\left(1 - \frac{c^2M^2\left(1-\frac{4M^2}{\ell^2}\right)^2}{\tilde{L}_n^2}\right). \quad (39)$$

Using $\hbar = r_c mc$ and Eq. (20) into above equation we obtain

$$E_n \approx mc^2\left(1 - \frac{M^2\left(1-\frac{4M^2}{\ell^2}\right)^2}{n^2r_c^2}\right). \quad (40)$$

The corresponding $(n+1)$th label energy can be written from the above Eq. (40) as

$$E_{n+1} \approx mc^2\left(1 - \frac{M^2\left(1-\frac{4M^2}{\ell^2}\right)^2}{r_c^2(n+1)^2}\right). \quad (41)$$

Therefore, the quantized energy difference between two nearby states of the form as

$\delta E = E_{n+1} - E_n$

$$\approx \frac{mc^2}{r_c^2}\left[\frac{1}{n^2}-\frac{1}{(n+1)^2}\right]M^2\left(1-\frac{4M^2}{\ell^2}\right)^2. \quad (42)$$

Neglecting the 4th and the higher powers of (M/ℓ) of the SAdS black hole radius, which gives Eq. (2) then we have

$$r_{SAdS} \approx 2M\left(1-\frac{4M^2}{\ell^2}\right). \quad (43)$$

We observe that when $\ell \to \infty$ then$r_{SAdS} \approx r_S$.Therefore, the Eq. (42) can be written for SAdS black hole

$$\delta E \approx \frac{mc^2 r_{SAdS}^2}{4r_c^2}\left[\frac{1}{n^2}-\frac{1}{(n+1)^2}\right]^2. \quad (44)$$

For large values of n, the bracket can be replaced by $2/n^3$ so that

$$\delta E \approx \frac{m^3c^4 r_{SAdS}^2}{2\hbar^2 n^3}=\frac{c^4m^3}{2\hbar^2 n^3}\times 4M^2\left(1-\frac{4M^2}{\ell^2}\right)^2 . \quad (45)$$

We observe that when n increase then δEdecreases. As $n \to \infty$ we have shown that the change of energy between two nearby states becomes zero.

5. Concluding Remarks

In this paper, we have presented the change of entropy for two nearby circular orbit around Schwarzschild Anti-de Sitter Black Hole by Energy Quantization process. We have also found the different energy labels ofSAdS black hole in nature can be performed in the same way as that for the electron signal inside the atom like Bohr's quantum theory and leads to the results on quantization of black hole [45].

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